SCIENCE

# Charlie Henry

## An analysis of Bell State Fidelity and its decay overtime

### REFLECTION STATEMENT

Throughout studying Physics, I found learning about quantum theory and its future impacts on society very interesting. In particular, I have become very hopeful and excited about the development of quantum computers, which will be able to break otherwise undecipherable codes, model global economies and improve weather forecasting in the near future. Given this, I took part in the UNSW SciX program to learn about the basic principles of quantum theory which apply to quantum computing. There, I focused on how the Bell State, the most simple and important algorithm, becomes more unreliable over time and affects all quantum computations.

In conducting an investigation, I initially reviewed the current literature on quantum computing and identified specific areas in which the current understanding of quantum computers can be improved. Then, I gathered my own data from a quantum computer in the US through writing code into the IBM Quantum Experience Platform and could analyse the data using Python code.

In essence, my work ‘An analysis of Bell State Fidelity and its decay overtime’ aims to estimate and provide reasons for unreliability in the base computations of a quantum computer which are key to its function. The hope is that my research could help scientists address the various problems inhibiting modern quantum computers and make them more reliable, so that they may achieve things never previously thought possible in a computer.

> “The hope is that my research could help scientists address the various problems inhibiting modern quantum computers and make them more reliable, so that they may achieve things never previously thought possible in a computer”

# REPORT

## The Abstract

In this investigation, a Bell State was constructed and conducted on the ‘ibmq_manila’ quantum computer and calculated using quantum state tomography to have fidelity 0.909 ±0.0132 and a statistically significant difference in the number of |01>, |10> and |11> results when compared to the ideal Bell State. The Bell State lifetimes were then increased by a varying number of identity gates and their fidelities were calculated on the same system. A Pearson’s $r^2$ value of 0.481 was obtained over very small increases of time, providing inconclusive results which were not statistically significant. However, over longer intervals of time, Pearson’s $r^2$ value of 0.959 was obtained suggesting a strong, negative linear correlation between the Bell State lifetime and its fidelity which was attributed to an increase in the relative phase of entangled qubits overtime and identity gate error.

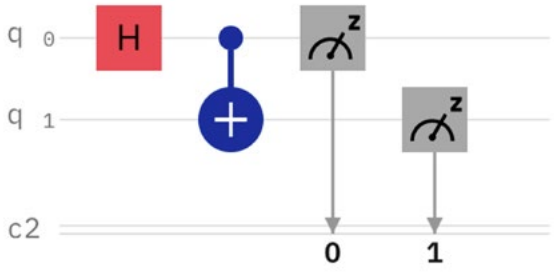
## Literature Review

### Quantum Computing and Bell States

Quantum computers can solve many problems which take too much time or memory on a regular computer. Fundamental to these different computers are qubits which have quantum mechanical properties (Gyongyosi & Imre, 2018). The two major principles, superposition and entanglement, have large applications in quantum error correction (Preskill, 1998), Shor’s integer factorisation and Grover’s search algorithms (Coles *et. al.*, 2018). However, quantum computers are error prone. For example, Grover’s algorithm was calculated to have a “65% success rate” on the IBM 5-qubit quantum computer which “is much lower” than classical computers operating at 100% (Coles *et. al.*, 2018 p. 10)

To conduct basic investigations on these root causes of algorithms, the lifetime of qubits in a basic Bell State can be analysed (Roos *et. al.*, 2004) by applying a Hadamard gate on qubit 0 before placing a CNOT gate on control qubit 0 and target qubit 1 and measuring both qubits as seen below in Figure 1.

**Right** Figure 1: Circuit Diagram of a Bell State



The aim of this literature review is to find the causes underpinning quantum computer infidelity reported in the literature, particularly relating to Bell States over time. It has found three major sources of error: measurement and quantum preparation error, decoherence and dephasing of entangled qubits and qubit energy-relaxation.

### State Preparation and Measurement Error

Using a Rydberg $C_Z$ gate to create a CNOT gate, a Bell-State was formed and was calculated to have fidelity $F_{BELL}=0.86$. (Graham *et. al.*, 2019) by considering the parity oscillations and results of the data, which is accurate but not as reliable as quantum state tomography. State Preparation and Measurement Error was found to occur due to atomic collisions occurring in finite vacuums and slight infidelity in the propagation of microwave pulses. However, this error was calculated as significantly smaller than qubit decoherence. The measurements taken to quantify sources of infidelity have no standard error reported and thus it is difficult to conclude how accurate these infidelity estimates are. Similar gate processes and error of this magnitude can be found in different silicon and superconducting quantum computers.

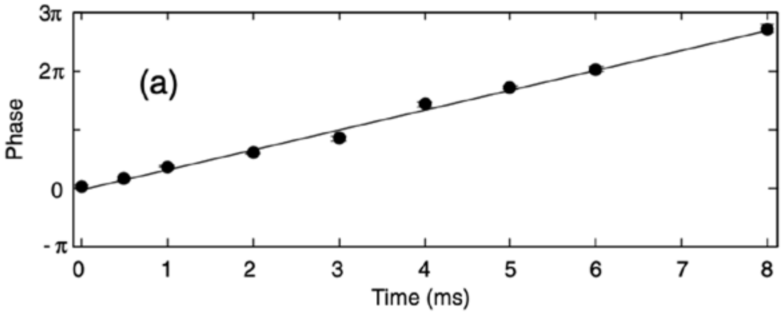
### Decoherence and Dephasing

Decoherence is the deterioration of a quantum state in which it loses its quantum properties (Cywinski *et. al.*, 2013). It is measured by, $T_2$, the time taken for a qubit to lose its superposition and $T_2^*$, the time take for multiple qubits to lose their superposition and is compared to $T_1$, the energy-relaxation time of a qubit (Wang, Zheng, Yin, 2008). It has been found that $T_2^*<T_2$, as $T_2^*$ considers potential space inhomogeneities that occurs between qubits. Generally, it was also found that the $T_2<T_1$, the pure energy-relaxation of qubits, as the $T_2$ time accounts for both energy-relaxation and quantum dephasing (Wang, Zheng, Yin, 2008). These arguments are contested by Burnett's experimental data (Burnett *et. al.*, 2019), but, in any case, fails to take account for the "continuous decay" (Cywinski *et. al.*, 2013, p. 1) of qubit decoherence by measuring it

as an interval of time. A qubit's progressive decohering over time, as well as differing quantum computers overtime, could be a potential reason for varying results in the field of these measures, with some ranging from nanoseconds to hundreds of milliseconds.

The causes of decoherence are found to be due to noise that occurs in the finite vacuum of a quantum computer. Superconducting qubits are affected by both Gaussian $1/f$ noise and Random Telegraph Noise, but mainly decohere due from the Gaussian $1/f$ noise which occurs due to the electric and magnetic dipoles fluctuating (Cywinski *et. al.*, 2013). In a different computer, however, it was found that superconducting qubits are limited by Lorentzian noise due to the two-level system nature of the qubit's spin (Burnett *et. al.*, 2019).

As seen in Figure 2, it has been found that decoherence leads to a difference in the phase between entangled qubits linearly, which was accurately measured using quantum state tomography. This creates inconsistency with the entanglement between the qubits and a source of infidelity in the quantum computer (Roos *et. al.*, 2004).



**Left** Figure 2: Linear relationship between time and the relative phase of two qubits entangled in a Bell State (Roos et. al., 2004).

Whilst the different lengths of decoherence and dephasing times are disputed, it is clear and accepted that decoherence occurs where qubits lose quantum properties overtime and is due to noise found within the quantum computer.

### Energy-Relaxation

In quantum computing, energy-relaxation is the phenomenon where the qubit has a tendency to return to a lower energy-state. In reliable and valid experimental data, it was found that relaxation occurs due to inconsistencies in the two-level system of a qubit, which fluctuates between energy states of spin up and spin down (Klimov *et. al.*, 2018). More specifically, in ion-trap computers, it was found that when the control atom of a two-qubit gate is in a high-energy state, there is an excess loss of energy in the target qubits. A delay was also discovered between the change in energy of a control and target qubit when using Rydberg pulses, which resulted in further energy loss (Maller *et. al.*, 2015).

Energy-relaxation is relevant because the energy transitions have been found to change the wave function of a qubit arrangement (Wang, Zheng, Yin, 2008), potentially leading to an incorrect result. Therefore, there is a valid argument that the best way to increase fidelity is to understand the inconsistencies in the two-level system of a quantum computer (Klimov *et. al.*, 2018). This is because these defects create energy-relaxation issues but, however, this argument overlooks concerns around the decoherence of qubits.

## Summary

Through surveying the literature, two major sources of quantum computing infidelity have been identified, energy-relaxation and decoherence of qubits, whilst there are other minor sources such as state preparation and measurement error. However, the magnitude of these effects is largely disputed and unclear for that of a simple Bell State and other circuits. Thus, further research is required into seeking how these errors effect the fidelity of quantum computers over longer time intervals.

## Scientific Research Question

How does an increase in lifetime of a constructed Bell State affect its fidelity?

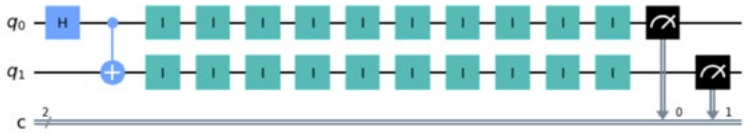
## Scientific Hypothesis

It is hypothesised that the fidelity of the Bell State decreases linearly with respect to time, and thus a strong negative correlation coefficient would be demonstrated between time and fidelity. This is due to the linear increase in relative phase between two entangled qubits overtime discovered through surveying the literature.

## Methodology

In this experiment, the independent variable was the lifetime of the Bell State, modified by the use of identity gates to extend the lifetime of the circuit, and the dependent variable was the circuit fidelity. As this was conducted remotely using the IBM Qiskit Lab, there were no ethical or safety issues in this investigation.

As seen in Figure 3, Bell Circuits were first prepared with CNOT and Hadamard gates before identity gates were placed on the circuit in order to increase the lifetime of the Bell State. This gate pattern ensured that there was no variable, such as the time or circuit creating the Bell State, which could potentially affect the final fidelity of the circuit.



**Left** Figure 3: Circuit diagram of a Bell State with 10 identity gates placed on each qubit.

After the circuits were created, further Qiskit Code was written to conduct the certain circuits on the quantum computer. There were two datasets collected, firstly of circuits with 0, 1, 2, 3 and 4 identity gates placed on each qubit and secondly with identity gates of 0, 10, 20, 30 and 40 to assess the impact of both shorter and longer intervals of time. For each respective dataset, 10 repetitions of the circuits were conducted all on the 'imbq_manila' computer with 8192 shots. As seen in Figure 4, transpiling circuits for the quantum computer was minimised in order to ensure that no circuits were changed except where absolutely necessary.

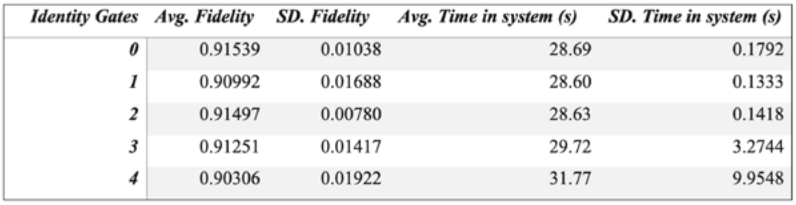
**Left** Figure 4: Circuit diagram of a Bell State with 0 identity gates conducted after being transpiled onto the 'ibmq_manila' system.

Quantum state tomography was used in order to ensure that phase rotations were considered in creating an accurate and reliable measure of fidelity. The jobs were saved so that the time and results of the circuit could be found and to confirm the correct circuits were run on the quantum system. The time, results and fidelity were then averaged for each set of 10 repetitions to eliminate the effect of outliers in the data and increase precision.

## Results

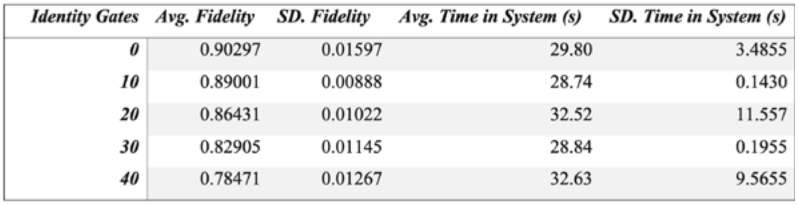
After the above method was conducted, the following measurements for time and fidelity were found for the respective circuits over the shorter and longer intervals.

**Right** Table 1: Time and fidelity measurements for circuits over shorter intervals of time.



| *Identity Gates* | *Avg. Fidelity* | *SD. Fidelity* | *Avg. Time in system (s)* | *SD. Time in system (s)* |
|---|---|---|---|---|
| *0* | 0.91539 | 0.01038 | 28.69 | 0.1792 |
| *1* | 0.90992 | 0.01688 | 28.60 | 0.1333 |
| *2* | 0.91497 | 0.00780 | 28.63 | 0.1418 |
| *3* | 0.91251 | 0.01417 | 29.72 | 3.2744 |
| *4* | 0.90306 | 0.01922 | 31.77 | 9.9548 |

**Right** Table 2: Time and fidelity measurements for circuits over longer intervals of time.

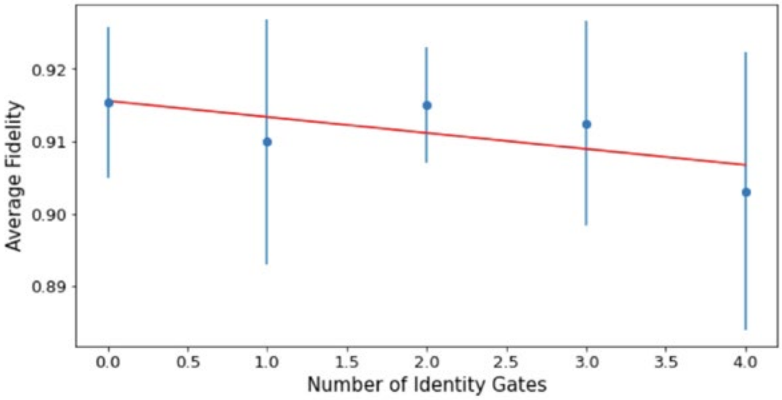


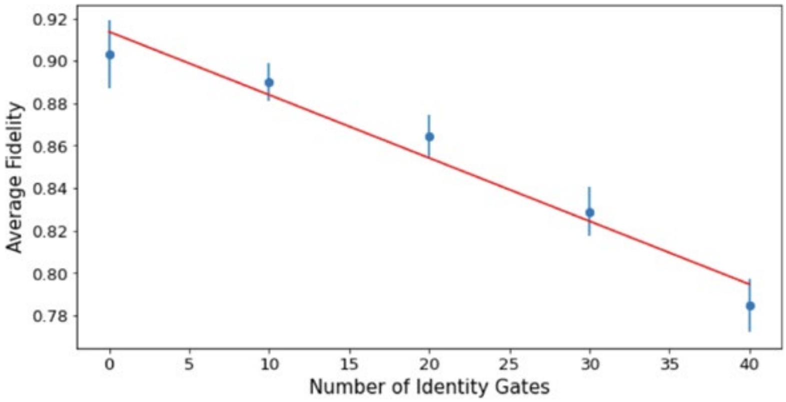
| *Identity Gates* | *Avg. Fidelity* | *SD. Fidelity* | *Avg. Time in System (s)* | *SD. Time in System (s)* |
|---|---|---|---|---|
| *0* | 0.90297 | 0.01597 | 29.80 | 3.4855 |
| *10* | 0.89001 | 0.00888 | 28.74 | 0.1430 |
| *20* | 0.86431 | 0.01022 | 32.52 | 11.557 |
| *30* | 0.82905 | 0.01145 | 28.84 | 0.1955 |
| *40* | 0.78471 | 0.01267 | 32.63 | 9.5655 |

In each dataset, it was found that the difference in the average time taken between the circuits was less than one standard deviation. Thus, it could be concluded that the average time in the quantum system was a measurement independent of the length of the circuits. Given this, lifetime of the Bell State was measured in identity gates.

The number of identity gates was graphed on the x-axis against the average fidelity of the circuit on the y-axis, with the standard deviation of measurements used as the standard error for the fidelity measurements as seen in Figures 5 and 6. Both the Pearson's correlation coefficient and the line of best fit y=mx+b were constructed for each dataset and are presented in Table 3. As r, the Pearson's correlation coefficient, only determines the strength of linear correlation between the variables, the coefficient was squared in order to attain a more accurate and reliable measure of the variance in fidelity which can be attributed to the change in identity gates.

**Right** Figure 5: Line of best fit y=mx+b between average fidelity and number of identity gates in the circuits over shorter time intervals.





**Left** Figure 6: Line of best fit y=mx+b between average fidelity and number of identity gates in the circuits used over longer time intervals

**Left** Table 3: Linear correlation and line of best fit parameters for each datase

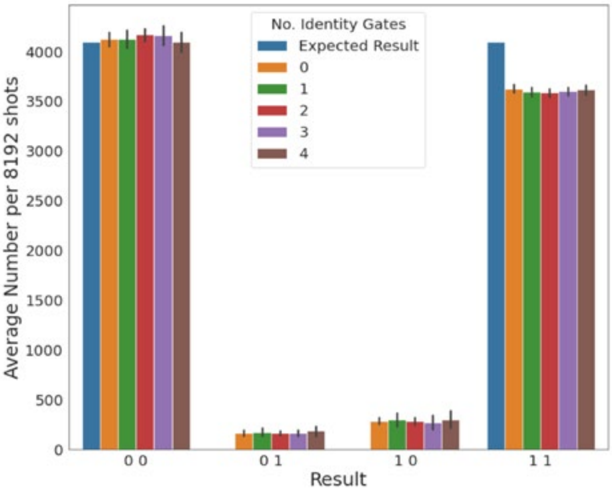
| | ***Shorter Time Intervals*** | ***Longer Time Intervals*** |
|---|---|---|
| ***r*** | -0.693 | -0.979 |
| ***$r^2$*** | 0.481 | 0.959 |
| ***m*** | $-2.208 \times 10^{-3}$ | $-2.975 \times 10^{-3}$ |
| ***m error*** | $1.325 \times 10^{-3}$ | $3.536 \times 10^{-4}$ |
| ***m error (%)*** | 60.0 | 11.9 |
| ***b value*** | 0.916 | 0.914 |
| ***b error*** | $3.245 \times 10^{-3}$ | $8.660 \times 10^{-3}$ |
| ***b error (%)*** | 0.354 | 0.948 |

From this data, it is unclear whether there is a clear, causal relationship between the fidelity of the quantum computer and the identity gates over very small intervals, or whether variations are due to chance. However, there is a much clearer relationship and causation between the lifetime of the Bell State and its decreasing fidelity over larger intervals of time. This is due to a decrease of $2.975\times10^{-3}\pm3.536\times10^{-4}$ in fidelity per identity gate.

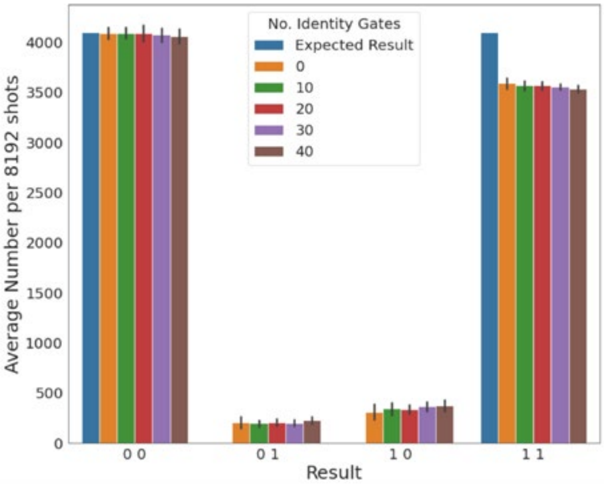
Through conducting the circuits, the difference in the average population distribution of the 8192 shots were measured for each dataset and compared to the expected values of 4096 |00> and 4096 |11> for a standard Bell State as seen in Figures 7 and 8.

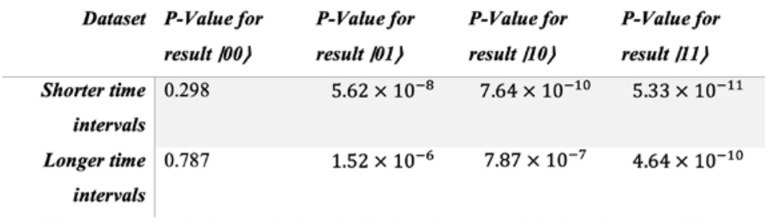
In each dataset, it was found that, for all results, the difference in the average number of shots was less than one standard deviation between average for the constructed circuits. Therefore, there is no statistical confidence that any difference between the distribution of results in the circuits is due to the lifetime of the Bell State.

**Right** Figure 7: Average population of the Bell State results for shorter time intervals. The standard error is the standard deviation in the results.



**Right** Figure 8: Average population of the Bell State results for longer time intervals. The standard error is one standard deviation in the results for each circuit.





| Dataset | P-Value for result $\|00\rangle$ | P-Value for result $\|01\rangle$ | P-Value for result $\|10\rangle$ | P-Value for result $\|11\rangle$ |
|---|---|---|---|---|
| ***Shorter time intervals*** | 0.298 | $5.62 \times 10^{-8}$ | $7.64 \times 10^{-10}$ | $5.33 \times 10^{-11}$ |
| ***Longer time intervals*** | 0.787 | $1.52 \times 10^{-6}$ | $7.87 \times 10^{-7}$ | $4.64 \times 10^{-10}$ |

**Left** Table 4: T-test results for population distributions between original Bell States with zero identity gates and expected results

Given the small amount of data used to determine the averages and standard error for each measurement, a two-sided t-test was conducted to analyse whether there were any statistically significant differences between result distributions for the original Bell State constructed with zero identity gates and the expected results in both datasets. The P-Value on the Student's curve with 9 degrees of freedom are shown in Table 4.

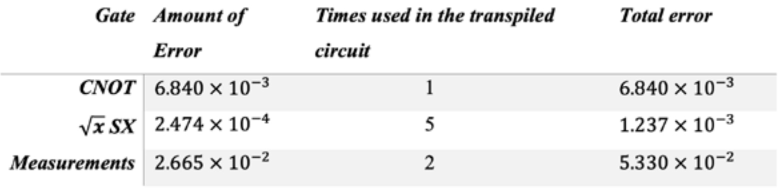
Taking a standard critical P-value of 0.05, it has been found that there is no statistical difference between the Bell States and the expected number of |00> results, but a statistically significant difference between the Bell State at the expected number of |01>, |10> and |11> results.

## Discussion

This experiment had procedural validity as it had one independent variable, the lifetime of the Bell State, and one dependent variable, the fidelity of the Bell State. All other variables were controlled, such as the computer used for each dataset and the time taken to create the Bell State before it underwent a series of identity gates. Each dataset was also collected in groups as measurements were taken in the shortest time possible, minimising any potential change in the quantum computing system overtime whether due to maintenance, recalibration, or deterioration. However, the measurements sometimes were taken over multiple hours due to waiting queues and the job limit held on the IBMQ platform and the 'imbq_manila' system.

Overall, this investigation was fairly reliable. The lifetime of the Bell State was measured by the identity gates in the circuit rather than the time of qubits in the system in order to obtain a reliable measure. The times, which were approximately around half a minute, would have taken into account the running of 9 programs per circuit for quantum state tomography, all of which were different lengths, measurement and state preparation time and thus could not be chosen as an accurate measure for the lifetime of the Bell State circuit. Given that the identity gate is an arbitrary wait gate, the average gate time as posted by IBM for the 'ibmq_manila' system, 368ns per gate (IBM, 2021), could not be applied as an estimate of time as that considers the time for microwave pulses to occur on any conventional gate. To enhance reliability and precision of results, 10 repetitions of each circuit were conducted and averaged to reduce the effect that outliers have on the data. However, upon evaluating the data, it became clear that the precision and reliability could be enhanced by taking in a much larger dataset. Due to the statistical phenomenon of regression to the mean (Galton, 1886), this would have likely resulted in a smaller standard deviation and error in the fidelity which diminished the statistical confidence of the findings, particularly for the analysis of fidelity results for shorter intervals of time and population distribution trends. A larger range of circuit lengths and identity gates could have also been used in order to provide more data points and obtain a more accurate line of best fit and correlation value. Therefore, whilst the results are fairly reliable in this investigation, a larger amount of data and range of circuits for each dataset would have enhanced the accuracy and statistical confidence in the trends found in the investigation.

**Right** Table 5: Gate Errors as reported by IBM and their total effect (IBM, 2021)



| *Gate* | *Amount of Error* | *Times used in the transpiled circuit* | *Total error* |
|---|---|---|---|
| ***CNOT*** | $6.840\times10^{-3}$ | 1 | $6.840\times10^{-3}$ |
| $\sqrt{x}$ ***SX*** | $2.474\times10^{-4}$ | 5 | $1.237\times10^{-3}$ |
| ***Measurements*** | $2.665\times10^{-2}$ | 2 | $5.330\times10^{-2}$ |

From the data in this experiment, it was found that Bell State has a source of error that produces a fidelity of approximately 0.909 ±0.0132 after averaging the fidelity results from both datasets for the plain Bell State with no identity gates. One source of infidelity for the computer is in the gate error for the 'ibmq_manila' system as seen in Table 5.

The RZ gate error could not be found for the computer on the IBM computer details but is likely to be similar in magnitude to √x SX as both gates are single qubit gates. Taking total error for the RZ gate to be that of the √x SX gate, $1.237\times10^{-3}$, the total gate and readout error is 0.0604, approximately 66.35% of the infidelity in the Bell State. Some of this gate infidelity is likely also due to qubit energy-relaxation effects when the rotations in the gates are applied through microwave pulses (Klimov *et. al.*, 2018)

Further infidelity in the data can be explained through considering the differences in the population distributions of the expected results and the Bell State. It was found that there was a significant difference between the |01> and |10> results in the Bell State when compared to the ideal Bell State, which indicates a loss of entanglement in the qubits. This is a problem consistent for other superconducting qubit systems (Garcia-Martin & Sierra, 2018), and occurs since qubit decoherence creates inconsistency in the entanglement of qubits in the quantum system (Cywinski *et. al.*, 2013). Since the |00> results remained similar to that of the predicted values, energy-relaxation phenomena in the qubits may have changed some |11> results into |01> or |10> results, also creating a significant loss of |11> results as seen in the data. This also explains why there is a larger proportion of |10> results than |01> results, as it is more likely for the target qubit of the CNOT gate to switch from a high to low energy state when the control qubit is initially in an excited state (Maller *et. al*., 2015).

Thus, any infidelity in the initial construction of the Bell State can be explained by sources of measurement or readout and gate error, which accounts for slightly under two-thirds of the error, as well as qubit decoherence and energy-relaxation when implementing the CNOT gate on the computer.

Any subsequent decay in smaller intervals of time were found to have been statistically insignificant due to limited repetition and a lack of precision in the data obtained from the experiment. Therefore, from the results in the investigation, it is unclear whether smaller changes in the lifetime of the Bell State cause a decrease in fidelity.

However, there was a decrease in fidelity over larger intervals of times for the Bell State, which was found to have a very strong linear correlation and $r^2$ value of 0.959. It was found that there was a decrease in fidelity of magnitude $2.975\times10^{-3}\pm3.536\times10^{-4}$ per identity gate placed on both qubits. One source of infidelity is found in the average identity gate error of the 'ibmq_manila' system. There is an error of $4.75\times10^{-4}$ per identity gate placed on both qubits (IBM, 2021), which accounts for approximately 16% of the overall decrease in the fidelity per identity gate and decreases linearly overtime. Since there was no significant difference in the distribution of results in the constructed circuits, the decrease in fidelity overtime time occurs due to a significant deterioration in the phase of the data (Graham *et al*., 2019). This relative dephasing between the two qubits has been found to increase linearly over time and cause a decrease in fidelity in the literature (Roos *et al*., 2004), which explains the high value for linear correlation in the investigation. Therefore, the statistically significant decrease in fidelity at a linear rate occurred as a result of the identity gate error in the quantum system and the relative dephasing between the entangled qubits.

Potential further research could involve taking greater datasets and increased precision to confirm that the patterns found in the longer time intervals are similar to those of smaller time intervals, or methods to minimise quantum infidelity in the Bell State. Future research could also be conducted into fidelity decay overtime with more useful and conventional gates in the quantum computer such as the X, Hadamard and Toffoli gates found in quantum computing algorithms.

### Conclusion

From the data, it was found that there was a strong negative correlation with $r^2=0.959$ between fidelity and Bell State lifetime over longer time intervals. Since it was found that there was no statistically significant difference in the population distribution over time in the circuits, the decrease in fidelity was explained by identity gate error and qubit dephasing overtime. However, due to imprecision and small datasets, there was uncertainty over whether the slight differences in fidelity and the population distribution over smaller time intervals was due to the changes in time or random error.

In the investigation, the fidelity in the computer's construction of the Bell State was calculated at 0.909 ±0.0132. After considering the differences in the results of the population distribution, it was found that gate and measurement error, qubit energy-relaxation and inconsistent entanglement were the major sources of infidelity in the Bell State.

### Acknowledgements

I would like to acknowledge the time and efforts of Samuel Wait, Georgina Carson and the UNSW SciX team for useful conversations around quantum computing theory, my experiment method and question. They also provided me with templates of Qiskit code which I could modify and change to run the experiment and analyse my data.

I would also like to thank Mr. Matthew Bentley, Mr. Robert Stuart and Dr. Laura McKemmish for their assistance in reviewing my work and providing me with feedback. I would also like to thank Ms. Leanne Gomez for proofreading my work.

## REFERENCE LIST

Burnett, J., Bengtsson, A., Scigliuzzo, M., Niepce, D., Kudra, M., Delsing, P. and Bylander, J., 2019. 'Decoherence benchmarking of superconducting qubits'. *npj Quantum Information*, vol. 5 no. 54.

Coles, P.J., Eidenbenz, S., Pakin, S., Adedoyin, A., Ambrosiano, J., Anisimov, P., Casper, W., Chennupati, G., Coffrin, C., Djidjev, H., Gunter, D., Karra, S., Lemons, N., Lin, S., Lokhov, A., Malyzhenkov, A., Mascarenas, D., Mniszewski, S., Nadiga, B., O'Malley, D., Oyen, D., Prasad, L., Roberts, R., Romero, P., Santhi, N., Sinitsyn, N., Swart, P., Vuffray, M., Wendelberger, J., Yoon, B., Zamora, R., and Zhu, W., 2018. 'Quantum Algorithm Implementations for Beginners'. *ArXiv*, vol. 1804.03719.

CywiĐski, Ł., Lutchyn, R., Nave, C. and Das Sarma, S., 2008. 'How to enhance dephasing time in superconducting qubits'. *Physical Review B*, vol. 77 no. 17.

Galton, F., 1886. ‘Regression Towards Mediocrity in Hereditary Stature’. *The Journal of the Anthropological Institute of Great Britain and Ireland*, vol. 15, p. 246.

García-Martín, D. & Sierra, G., 2018. ‘Five Experimental Tests on the 5-Qubit IBM Quantum Computer’. *Journal of Applied Mathematics and Physics*, vol. 6 no. 7, pp. 1460-1475.

Graham, T., Kwon, M., Grinkemeyer, B., Marra, Z., Jiang, X., Lichtman, M., Sun, Y., Ebert, M. and Saffman, M., 2019. ‘Rydberg-Mediated Entanglement in a Two-Dimensional Neutral Atom Qubit Array’. *Physical Review Letters*, vol. 123 no. 23.

Gyongyosi, L., Imre, S., 2019, ‘A Survey on quantum computing technology’, *Computer Science Review*, Vol. 31, pp. 51-71.

IBM, 2021. ‘Quantum Services’. *IBM Quantum*, viewed 11 July 2021, https://quantum-computing.ibm.com/services?services=systems.

Klimov, P., Kelly, J., Chen, Z., Neeley, M., Megrant, A., Burkett, B., Barends, R., Arya, K., Chiaro, B., Chen, Y., Dunsworth, A., Fowler, A., Foxen, B., Gidney, C., Giustina, M., Graff, R., Huang, T., Jeffrey, E., Lucero, E., Mutus, J., Naaman, O., Neill, C., Quintana, C., Roushan, P., Sank, D., Vainsencher, A., Wenner, J., White, T., Boixo, S., Babbush, R., Smelyanskiy, V., Neven, H. and Martinis, J., 2018. ‘Fluctuations of Energy-Relaxation Times in Superconducting Qubits’. *Physical Review Letters*, vol. 121 no. 9.

Maller, K., Lichtman, M., Xia, T., Sun, Y., Piotrowicz, M., Carr, A., Isenhower, L. and Saffman, M., 2015. ‘Rydberg-blockade controlled-not gate and entanglement in a two-dimensional array of neutral-atom qubits’. *Physical Review A*, vol. 92 no. 2.

Preskill, J., 1998, ‘Reliable quantum computers’. *Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences*, Vol. 454 No. 1969, pp. 385-410.

Roos, C., Lancaster, G., Riebe, M., Häffner, H., Hänsel, W., Gulde, S., Becher, C., Eschner, J., Schmidt-Kaler, F. and Blatt, R., 2004. ‘Bell States of Atoms with Ultralong Lifetimes and Their Tomographic State Analysis’. *Physical Review Letters*, vol. 92 no. 22.

Wang, X., Zheng, Y. and Yin, S., 2005. ‘Spin relaxation and decoherence of two-level systems’. *Physical Review B*, vol. 72 no.12.